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Influence of azimuthal anchoring on the structure and range of occurrence of spontaneous periodic deformations in hybrid aligned nematic layers

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Periodic deformations spontaneously arising in hybrid aligned nematics are studied numerically. The so-called splay stripes are considered for material parameters close to those known for 5CB and for the saddle–splay elastic constant k_{24} values allowed by the Ericksen inequalities. The role of finite azimuthal anchoring on the planar boundary surface is investigated. The director distribution is determined for various azimuthal anchoring parameters. It is found that azimuthal anchoring suppresses the periodic deformations, damping their amplitude and decreasing all the parameter ranges for which the stripes exist. For the material constants considered, this effect is more pronounced for stripes which are realized for a negative k_{24} elastic constant, than for stripes arising if $k_{24}>0$.

1. Introduction

The so-called hybrid aligned structure (HAN) typically occurs in a nematic layer when the boundary anchoring conditions induce planar alignment on one surface and homeotropic alignment on the other. The director distribution is distorted with comparable contributions of splay and bend. However, if the layer is thinner than the critical value d_{H}, the director field becomes uniform [1]. The uniform distribution can be planar (when the planar polar anchoring prevails) or homeotropic (when the homeotropic polar anchoring dominates).

In 1990, another possibility was found experimentally by Lavrentovich and Pergamenshchik in thin layers with a hybrid orientation [2]. A nematic liquid crystal was deposited on the surface of glycerin with the upper surface free. The director alignment was planar at the liquid crystal–glycerin interface and nearly homeotropic at the free surface. The stripes, visible under a polarizing microscope, revealed a periodically deformed structure. Similar periodic patterns were observed in layers with pure planar [3] and pure homeotropic [4] surface conditions and in twisted structures [5] under the influence of external fields. In the hybrid case, however, stripes can arise spontaneously, even without action of a field.

This effect was theoretically studied in depth by Lavrentovich and Pergamenshchik [6–8] and others [9–15]. Two types of periodic structure were predicted: splay stripes, when planar anchoring is stronger than homeotropic anchoring, and bend stripes in the opposite case. This paper, as well as most of the earlier papers, is devoted to splay stripes observed by Lavrentovich and Pergamenshchik. For suitable elastic properties of the nematic and proper anchoring conditions, splay stripes appear if the thickness of the layer is larger than d_{P} and lower than d_{A}, where $d_{\mathrm{P}}<d_{\mathrm{H}}$ and $d_{\mathrm{A}}>d_{\mathrm{H}}$ [2, 9, 14, 15]. A significant influence of the k_{24} elastic constant on the appearance and structure of the splay stripes was found [7, 11]. Our earlier calculations showed that two different periodic structures (mode 1 and mode 2) appeared depending on the $k_{\mathrm{s}}=k_{24}/k_{11}$ value [15]. Most of the earlier studies were based on linear analysis, and so concerned small deformations in the vicinity of d_{P} [2, 6–13]. The values of d_{p} and their dependence on material and layer parameters were found. The values of d_{A} were calculated only in a few studies [14–16].

Anchoring conditions play a very important role in the occurrence of splay stripes. Periodic deformations arise when the polar anchoring energy on the planar plate is finite and larger than on the opposite plate. Lavrentovich and Pergamenshchik [2] assumed the azimuthal degeneration of anchoring due to the isotropic character of the ambient medium. The same assumption was made in our earlier work [15, 16]. Studies in which non-zero azimuthal anchoring was

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taken into account, restricted the considerations to the vicinity of d_P [9–12].

This paper is devoted to numerical investigations of the influence of non-zero azimuthal anchoring conditions on the structure and occurrence of spontaneous splay stripes in hybrid aligned nematic layers. We calculate the director distributions for various anchoring conditions. We find the influence of azimuthal anchoring at the planar boundary surface on the ranges of parameters essential for the existence of the stripes, i.e. on the thickness of the layer, elastic properties of the nematic, and polar anchoring conditions.

In the following section, details of the system under consideration are given and the method of computation is briefly described. The results are presented in § 3, while § 4 is devoted to a short discussion.

2. Method

Our numerical calculations, performed within the framework of continuum theory, consider an infinite nematic layer of thickness d placed between two plates parallel to the xy-plane and positioned at $z=\pm d/2$. Planar alignment was assumed at $z=-d/2$ and homeotropic alignment at $z=d/2$. The two angles required to describe the director distribution were θ measured between $\mathbf{n}$ and the xy-plane and φ measured between the x-axis and the projection of $\mathbf{n}$ on the xy-plane. The stripes were directed along the x-axis. The angles θ and φ were dependent on y and z. The y-dependence was periodic. The periodicity can be described by the wave vector $\mathbf{q}\|y$ or by the spatial period $\lambda=2\pi/q$ where $q=|\mathbf{q}|$.

The method was based on numerical minimization of the free energy F of a single stripe per unit area of the layer expressed by:

$$F=\frac{k_{11}}{2d}\int_0^1\left\{\int_{-1/2}^{1/2}\left\{(\nabla\mathbf{n})^2+k_t[\mathbf{n}\cdot(\nabla\times\mathbf{n})]^2+k_b[\mathbf{n}\times(\nabla\times\mathbf{n})]^2\right.\right.$$
$$\left.\left.+2(k_t+k_s)\nabla\cdot[\mathbf{n}(\nabla\cdot\mathbf{n})+\mathbf{n}\times\nabla\times\mathbf{n}]\right\}\mathrm{d}\zeta+f_P+f_H\right\}\mathrm{d}\eta \quad (1)$$

where

$$f_P=-\gamma_{\theta P}\cos^2\theta(\eta,-1/2)-\gamma_{\varphi P}\cos^2\varphi(\eta,-1/2) \quad (2)$$

and

$$f_H=-\gamma_{\theta H}\sin^2\theta(\eta,1/2) \quad (3)$$

are the surface densities of free energy (indices P and H denote planar and homeotropic plates, respectively); $\gamma_{\theta P}=W_{\theta P}d/k_{11}$, $\gamma_{\theta H}=W_{\theta H}d/k_{11}$ and $\gamma_{\varphi P}=W_{\varphi P}d/k_{11}$ are the polar and azimuthal anchoring parameters, respectively and $W_{\theta P}$, $W_{\theta H}$ and $W_{\varphi P}$ are the anchoring strength parameters, $k_b=k_{33}/k_{11}$, $k_t=k_{22}/k_{11}$ and $k_s=k_{24}/k_{11}$ are the reduced elastic constants, $\eta=y/\lambda$ and $\zeta=z/d$ denote the reduced co-ordinates and $\nabla=\left(\frac{d}{\lambda}\frac{\partial}{\partial\eta},\frac{\partial}{\partial\zeta}\right)$. In this way the free energy density per unit area of the layer F can be expressed in units equal to k_{11}/d. Periodic boundary conditions along the y-axis were imposed. The functions $\theta(\eta,\zeta)$ and $\varphi(\eta,\zeta)$ were approximated by discrete values of the angles. The energy (1) was then minimized in the manner described in detail in our earlier work [15, 17].

The elastic constant ratios used for computations were chosen to be close to those of 5CB which was used by Lavrentovich and Pergamenshchik in their experiment: $k_b=1.3$, $k_t=0.5$ [18, 19]. The values of k_s were varied throughout the entire allowed range $(-0.5, 0.5)$ resulting from Ericksen inequalities [20]:

$$\begin{array}{c} k_s\leqslant 2-k_t \\ -k_t\leqslant k_s\leqslant k_t \end{array}. \quad (4)$$

The other surface-like elastic constant, k_{13}, was assumed to be zero, according to the theoretical result obtained by Yokoyama [21].

The calculations were performed for various values of $\gamma_{\theta P}$, $\gamma_{\theta H}$ and $\gamma_{\varphi P}$. The relation $\gamma_{\theta P}=2\gamma_{\theta H}$ was adopted in all calculations. For mode 1, azimuthal anchoring energy on the planar plate was assumed to be one order of magnitude smaller than polar energy, which is the typical relationship [22, 23]. For mode 2, values larger by one order of magnitude were chosen. Azimuthal anchoring energy on the homeotropic plate was always equal to zero.

3. Results

Two modes with different periodic structures were recognized in hybrid aligned nematic layers, depending on the k_s value. When k_s is lower than a critical value k_{sC1}, mode 1 appears, whereas mode 2 is realized when k_s is higher than some other critical value k_{sC2}. For conical degeneration and for $k_t=0.5$ used in our calculations, both critical values were zero: $k_{sC1}=k_{sC2}=0$. This means that mode 1 is predicted for negative k_s, and mode 2 for positive k_s.

Periodic deformations are strongly suppressed by azimuthal anchoring. Non-zero azimuthal anchoring influences both modes, but mode 1 is evidently affected more. For the sake of clarity, the influence on each mode will be presented separately.

3.1. *Mode 1*

The director distribution is strongly dependent on azimuthal anchoring. This effect can be illustrated by the decrease in the amplitudes of the $\theta(\eta, 0)$ and $\varphi(\eta, 0)$ functions with azimuthal anchoring on the planar plate $\gamma_{\varphi P}$. The corresponding dependences are shown in figure 1, where the amplitudes θ_m and φ_m are plotted as functions of $\gamma_{\varphi P}$ for $\gamma_{\theta P}=1$. Obviously, the influence of finite azimuthal energy on the angle φ is more

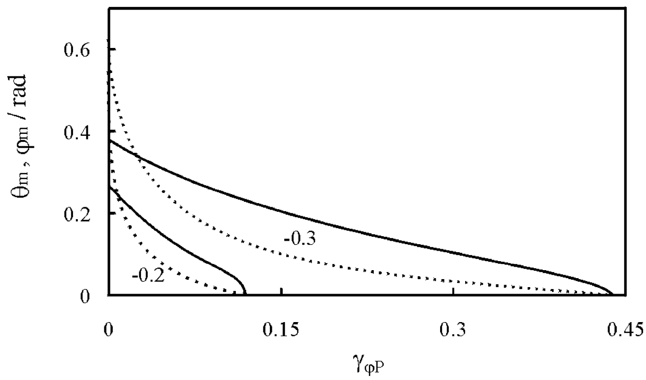

Figure 1. Amplitudes θ_{m} (solid line) and φ_{m} (dotted line) plotted as functions of $\gamma_{\varphi\mathrm{P}}$ for $k_{\mathrm{s}}=-0.3$ and $k_{\mathrm{s}}=-0.2$. Mode 1: $\gamma_{\theta\mathrm{P}}=1$; $\gamma_{\theta\mathrm{H}}=0.5$.

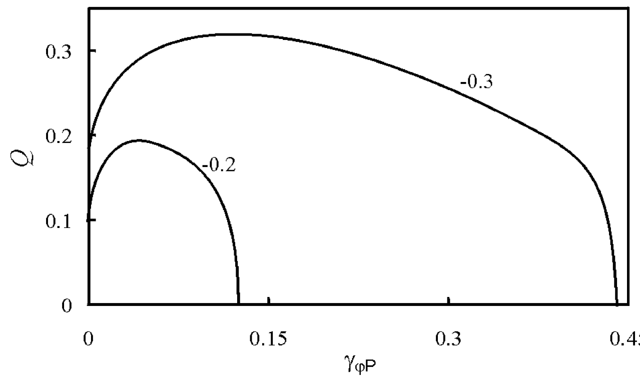

Figure 3. The dependences of dimensionless wave number Q of mode 1 on $\gamma_{\varphi\mathrm{P}}$ for $k_{\mathrm{s}}=-0.3$ and $k_{\mathrm{s}}=-0.2$; $\gamma_{\theta\mathrm{P}}=1$ and $\gamma_{\theta\mathrm{H}}=0.5$.

pronounced. Sufficiently large azimuthal anchoring can totally suppress the stripes.

The width of the stripe is also influenced by azimuthal anchoring. In the following, this effect is presented by means of the reduced wave number $Q=qd$. In the case of azimuthal degeneration, the stripes appear when $\gamma_{\theta\mathrm{P}}$ is greater than the lower critical value γ_{C1}. In the limit $\gamma_{\theta\mathrm{P}}\rightarrow\gamma_{\mathrm{C1}}$, the wave number of the periodic distortion and its amplitude tend to zero. For some range of $\gamma_{\theta\mathrm{P}}$, the wave number of the periodic distortion increases with $\gamma_{\theta\mathrm{P}}$ and for higher $\gamma_{\theta\mathrm{P}}$ it decreases again (figure 2). The amplitude of distortion increases with $\gamma_{\theta\mathrm{P}}$. When $\gamma_{\theta\mathrm{P}}$ exceeds another critical value $\gamma_{\theta2}$, periodic deformations become energetically unfavourable and are replaced by homogeneous hybrid alignment, shown in figure 2 by means of the dashed part of curve 1.

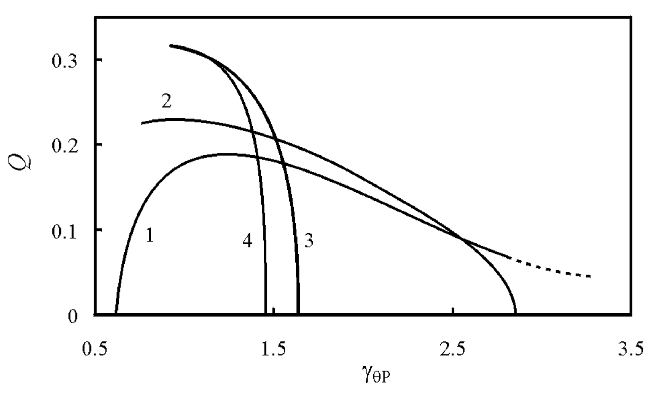

Figure 2. Dimensionless wave number $Q=qd$ of mode 1 as a function of $\gamma_{\theta\mathrm{P}}$. Curve 1: $\gamma_{\varphi\mathrm{P}}=0$ (the dashed part of this curve corresponds to the energy of a periodically deformed layer larger than the energy of the homogeneous structure); curve 2: $\gamma_{\varphi\mathrm{P}}=0.01$; curve 3: $\gamma_{\varphi\mathrm{P}}=0.1$; curve 4: $\gamma_{\varphi\mathrm{P}}=0.1\gamma_{\theta\mathrm{P}}$; $\gamma_{\theta\mathrm{H}}=\gamma_{\theta\mathrm{P}}/2$ and $k_{\mathrm{s}}=-0.3$ in all cases.

When azimuthal anchoring is non-zero, the dependence of the wave number on $\gamma_{\theta\mathrm{P}}$ is different. Stripes appear with finite wave number at γ_{C1}. When $\gamma_{\theta\mathrm{P}}$ is increased, the wave number decreases. Simultaneously, the amplitude of the deformation becomes larger. For $\gamma_{\theta\mathrm{P}}$ approaching some other critical value γ_{C2}, the wave number decreases rapidly to zero (figure 2, curves 2–4). Such a form of the dependence of Q on a control parameter is characteristic also for other pattern-forming systems [15–17, 24]. The decay is particularly sharp when $\gamma_{\varphi\mathrm{P}}$ increases with $\gamma_{\theta\mathrm{P}}$ according to assumption $\gamma_{\varphi\mathrm{P}}=0.1\gamma_{\theta\mathrm{P}}$ (curve 4).

The role of the azimuthal anchoring parameter $\gamma_{\varphi\mathrm{P}}$ is also shown in figures 3 and 4. In both figures $\gamma_{\theta\mathrm{P}}=1$. In figure 3, the dependence of the wave number on $\gamma_{\varphi\mathrm{P}}$ is plotted. Again, two ranges of the azimuthal anchoring parameter can be distinguished: one in which the wave number increases with $\gamma_{\varphi\mathrm{P}}$, and another in which Q decreases with $\gamma_{\varphi\mathrm{P}}$. The narrowest stripes occur when

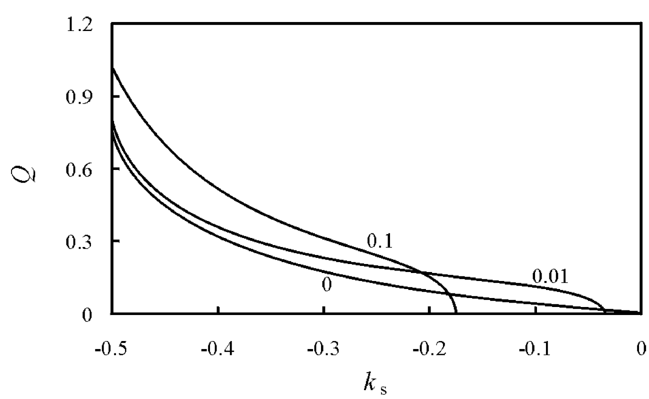

Figure 4. Dimensionless wave number Q of mode 1 as a function of k_{s} for different azimuthal anchoring $\gamma_{\varphi\mathrm{P}}$ energies indicated by curves: in all cases $\gamma_{\theta\mathrm{P}}=1$ and $\gamma_{\theta\mathrm{H}}=0.5$.

the azimuthal anchoring energy is about one order of magnitude smaller than the polar anchoring energy on the planar plate. When $\gamma_{\varphi P}$ approaches some critical value, the wave number tends rapidly to zero. Simultaneously the amplitude of the deformation decays as shown in figure 1. Higher $\gamma_{\varphi P}$ excludes the appearance of the stripes. In figure 4, the $Q(k_s)$ dependences for different anchoring conditions are given. The wave number decreases monotonically with k_s and goes to zero at k_{sC1}. A significant decrease of k_{sC1} with azimuthal anchoring is evident.

When $\gamma_{\theta P}$ tends to γ_{C2}, a stripe become wider and regions with nearly homogeneous hybrid alignment of opposite senses arise in both halves of a stripe. These regions widen rapidly to infinity at γ_{C2}, which means that for $\gamma_{\theta P}>\gamma_{C2}$ the layer adopts the homogeneous hybrid structure. These effects are illustrated in figure 5 which shows the $\theta(y)$ profiles for several values of $\gamma_{\theta P}$ in a layer with non-zero azimuthal anchoring.

The range of k_s, as well as the range (γ_{C1}, γ_{C2}) for which the stripes exist, narrow with γ_{φ}. This effect is illustrated in figure 6 where the regions of existence of stripes in the plane (k_s, $\gamma_{\theta P}$) are shown. It is evident that the upper boundaries of these regions are more strongly influenced by $\gamma_{\varphi P}$ than the lower. The range of $\gamma_{\theta P}$ in which stripes appear decreases with k_s in each case. When k_s tends to the critical value k_{sC1} from below, the amplitude of deformation is damped (and simultaneously the wave number decreases to zero, see figure 4). Sufficiently large azimuthal anchoring excludes the occurrence of stripes of mode 1, i.e. the critical quantities tend to their limiting values: $k_{sC1}\rightarrow -0.5$, $\gamma_{C1}\rightarrow\gamma_{C2}\rightarrow 1$ (for $k_t=0.5$ and $\gamma_{\theta P}/\gamma_{\theta H}=2$).

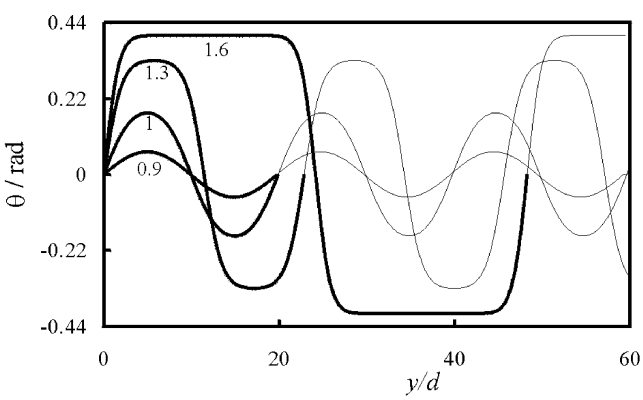

Figure 5. Dependences of $\theta(y)$ plotted for characteristic cross-sections $\zeta=0$ for several values of $\gamma_{\theta P}$ (indicated for each curve), bold sections of the curves correspond to single stripes. Mode 1: $k_s=-0.3$, $\gamma_{\varphi P}=0.1$, $\gamma_{\theta H}=0.5\gamma_{\theta P}$.

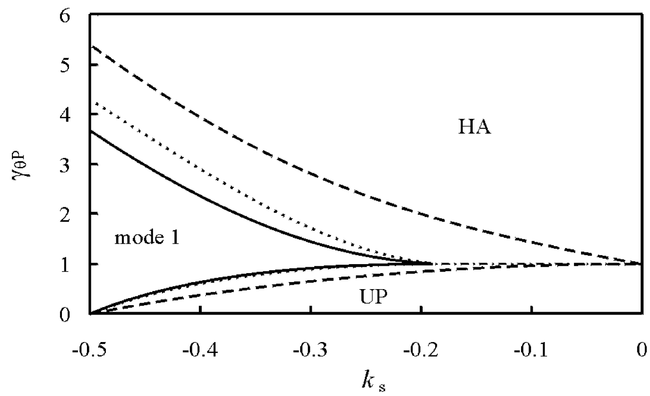

Figure 6. The ranges of parameters k_s and $\gamma_{\theta P}$ assuring occurrence of stripes of mode 1 for different anchoring conditions: dashed line denotes conical degeneration case; dotted line denotes $\gamma_{\varphi P}=0.1$; solid line denotes $\gamma_{\varphi P}=0.1\gamma_{\theta P}$; chain line denotes the boundary between planar and homogeneous hybrid alignments in the absence of stripes; HA = homogeneous hybrid alignment; UP = uniform planar alignment.

3.2. *Mode 2*

The influence of azimuthal anchoring on mode 2 of the splay stripes is much weaker than in the case of mode 1. The amplitudes θ_m and φ_m are only slightly lowered relative to their values obtained for the conical degeneration case. The most pronounced effect is the decrease of the angle φ on the planar plane. Simultaneously, only a marginal change of φ on the homeotropic plane was found. Even unusually large planar azimuthal anchoring causes only small changes of the angle θ.

In the layer without azimuthal anchoring, stripes appear with finite wave number for arbitrarily low $\gamma_{\theta P}$ (however, for small k_s, there exist two critical values γ_{C1}' and γ_{C1}'', between which the planar orientation occurs). For small $\gamma_{\theta P}$, the wave number of distortion increases slightly with $\gamma_{\theta P}$. For $\gamma_{\theta P}$ approaching γ_{C2}, the wave number decreases rapidly to zero, which means that the stripe period tends to infinity. The same effects were found in the case of non-zero azimuthal anchoring conditions, but the values of γ_{C2} were smaller. The dependences of the wave numbers on $\gamma_{\theta P}$ for various azimuthal anchoring conditions are shown in figure 7.

The rapid increase of the stripe period to infinity is accompanied by the arising of nearly homogeneous regions in each half of a stripe, as in the case of mode 1. For $\gamma_{\theta P}>\gamma_{C2}$, the homogeneous hybrid alignment is realized. The amplitude of the distortion increases with $\gamma_{\theta P}$.

In the case of finite azimuthal anchoring conditions as well as in the case of azimuthal degeneration, the amplitude of the distortion and wave number decay to

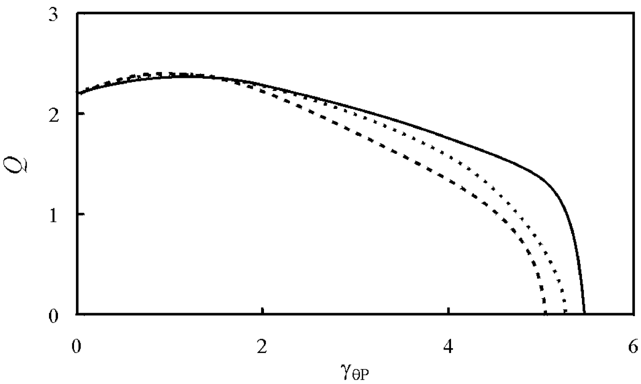

Figure 7. Dimensionless wave number Q of mode 2 as a function of $\gamma_{\theta P}$: solid line denotes the azimuthal degeneration case; dotted line denotes $\gamma_{\varphi P}=1$; dashed line denotes $\gamma_{\varphi P}=\gamma_{\theta P}$; $k_s=0.25$.

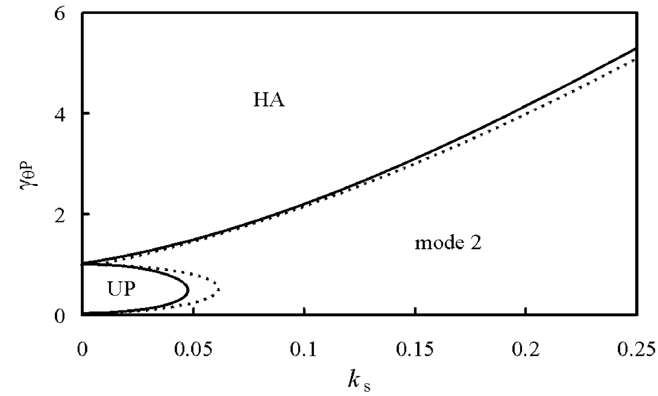

Figure 9. The ranges of parameters k_s and $\gamma_{\theta P}$ assuring occurrence of stripes of mode 2 for zero (solid line) and non-zero ($\gamma_{\varphi P}=\gamma_{\theta P}$; dotted line) azimuthal anchoring conditions; HA = homogeneous hybrid alignment; UP = uniform planar alignment.

zero when k_s is decreased. The dependences of the wave numbers on k_s for zero and non-zero azimuthal anchoring conditions are shown in figure 8. A slight increase of the critical value k_{sC2} is observed as a consequence of azimuthal anchoring.

In figure 9, the ranges of occurrence of mode 2 in the plane (k_s, $\gamma_{\theta P}$) are shown by the plots of γ_{C2}, γ_{C1}' and γ_{C1}'' as functions of k_s for zero and non-zero azimuthal anchoring conditions. A slight decrease of γ_{C2} with azimuthal anchoring is evident. Simultaneously, the range of existence of the planar orientation is enlarged.

4. Discussion

Periodic deformations in hybrid nematic layers are fairly complex phenomena. Their features depend on many parameters—three elastic constant ratios k_b, k_t and k_s, thickness of the sample and anchoring conditions characterized by four coefficients. In this paper, we present the results of numerical calculations which allow us to establish the influence of azimuthal anchoring on the structure of splay stripes and on the conditions for their existence. We focused on a single typical liquid crystal, 5CB, in which periodic deformations were observed, and used its material parameters. Since the value of the surface-like elastic constant k_{24} is unknown, values from the whole range determined by the Ericksen inequalities were taken.

The method of our computations is based on the iterative minimization of the total free energy of the layer. During this process, the director orientation is varied by small intervals which may be interpreted as introducing a small perturbation. The configuration corresponding to the lowest energy is accepted after each iteration. The solutions obtained in this way are due to the minimum of the free energy and in consequence are stable against small perturbations. This applies to all the states distinguished in our diagrams (figures 6 and 9). For this reason we have omitted additional stability analysis.

Azimuthal anchoring in a hybrid cell has physical sense only at the planar plate. The azimuthal anchoring energy can be zero as well as having a finite value. The former takes place when the planar background is isotropic. Standard methods of planar alignment such as rubbing or SiO evaporation make the substrate anisotropic, so in consequence the azimuthal anchoring energy is non-zero. Its typical values are one or two orders of magnitude smaller than the polar anchoring energy. Such values were taken into account in the case of mode 1. In the case of mode 2, such typical values have a very small influence. Therefore exaggerated values of azimuthal anchoring energies (i.e. comparable with the polar anchoring energy) were used to

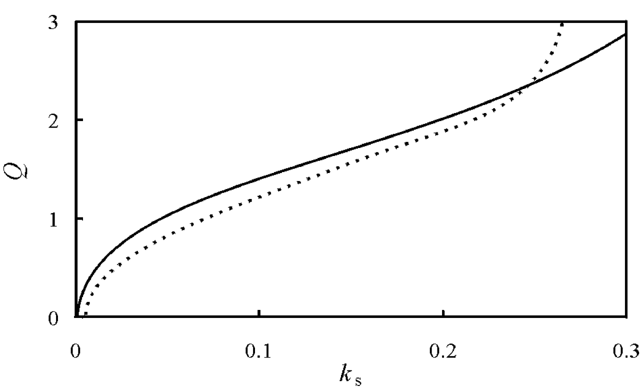

Figure 8. Dimensionless wave number Q of mode 2 as a function of k_s for zero (solid line) and non-zero ($\gamma_{\varphi P}=\gamma_{\theta P}$; dotted line) azimuthal anchoring conditions; $\gamma_{\theta P}=1$ and $\gamma_{\theta H}=0.5$.

demonstrate the character of possible changes of the stripes. It seems that there is no possibility of the effective damping of the stripes of mode 2 by imposing azimuthal anchoring energy of any realistic magnitude.

We found that the influence of azimuthal anchoring energy on both modes was qualitatively similar, but the damping of mode 1 was more pronounced. The main consequences of finite azimuthal anchoring conditions can be summarized thus:

(i) The amplitudes of angles θ and φ decrease with azimuthal anchoring. In the case of mode 1, the increase of γ_φ causes significant decrease of both angles, but the decrease of φ is more pronounced. Sufficiently large $\gamma_{\varphi P}$ values eliminate the stripes. In the case of mode 2, the most significant consequence of increasing of γ_φ is the decrease of angle φ on the planar plate. Any decrease of angle θ on both plates and of angle φ on the homeotropic plane is rather insignificant.

(ii) Under non-zero azimuthal anchoring conditions, the stripes of mode 1 appear with finite wave number at $\gamma_{\theta P}=\gamma_{C1}$. The values of γ_{C1} coincide with the values calculated for mode 1 in earlier work [11]. At $\gamma_{\theta P}=\gamma_{C2}$, the wave number rapidly decreases to zero, which means that above γ_{C2} the sample has homogeneous hybrid structure. These results are in contrast to the case of azimuthal degeneration when the wave number tends to 0 at γ_{C1} and remains finite at γ_{C2}.

(iii) Under finite azimuthal anchoring conditions, a range (k_{sC1}, k_{sC2}) of k_s appears in which the stripes of any mode are excluded. It widens with the azimuthal anchoring strength.

(iv) The critical values γ_{C1} and γ_{C2} plotted in the (k_s, $\gamma_{\theta P}$) plane for different azimuthal anchoring conditions indicate that the regions of existence of the splay stripes decrease with azimuthal anchoring.

The disappearance and reappearance of mode 2 seen in figure 9 is caused by the delicate interplay between various components of the free energy and can be interpreted as follows. The region of existence of the stripes of mode 1 (occurring for for $k_s<0$), plotted in the (k_s, $\gamma_{\theta P}$) plane, has a characteristic wedge-like shape which was found also for other periodic deformations in liquid crystal layers [14, 17, 24, 25]. In particular, mode 1 stripes disappear if $\gamma_{\theta P}$ is too low and are replaced by the uniform planar state which exists in a significant region of the (k_s, $\gamma_{\theta P}$) plane. One might suppose that a similar wedge-like region should exist for $k_s>0$ when mode 2 is realized. However, the mode 2 region also spreads over the arbitrarily low $\gamma_{\theta P}$ and reduces the region of the uniform planar state to the small area denoted by UP in figure 9. Four components of the total free energy of the layer should be compared to explain these effects:

(a) the anchoring energy at the planar plate is negative and minimum in the UP state and negative but greater than the minimum value in the periodically deformed state;
(b) the anchoring energy at the homeotropic plate is zero in the UP state and negative in the periodic state;
(c) the term containing the coefficient $k_{24}+k_{22}$ is zero in the UP state and negative in periodically deformed structures;
(d) the elastic term is zero in the UP state and positive in the periodic state.

The periodic pattern has lower energy than the UP state if the negative contributions (b) and (c) compensate its elastic energy (d) plus the positive contribution equal to the difference of the anchoring energies (a) of the uniform planar and periodic states. When $k_{24}<0$, the coefficient $k_{24}+k_{22}$ has a relatively small magnitude (between 0 and 0.5 in our case) and the compensation does not occur. In contrast, when $k_{24}>0$, the coefficient is larger (it ranges from 0.5 to 1) and the contributions (c) together with (b) give rise to the periodic pattern of mode 2, especially when $\gamma_{\theta P}$ is low, giving only a small excess above the minimum of the planar anchoring energy. In consequence, the UP state region becomes surrounded by the extended mode 2 region.

To our knowledge, the only experimental evidence of the spontaneous periodic pattern in HAN was obtained in the absence of azimuthal anchoring [2, 8, 14], so we are unable to compare our calculated results with experimental data. Nevertheless, some other aspects of our results agree qualitatively with the facts reported in [2, 8, 14]. In particular, the measured period of the stripes is comparable with the period calculated for mode 1. It increases with the thickness of the layer which is also predicted by our work.

As our additional calculations with use of Mueller matrix method [26, 15] have shown, the damping of the amplitudes of deformation of mode 1 due to the strong influence of azimuthal anchoring causes a drastic decrease of the visibility of the stripes. This fact as well as significant limitations of the sets of k_s and $\gamma_{\theta P}$ parameters for which the stripes occur, make the experimental observation of them difficult in real hybrid aligned nematic layers with no azimuthal degeneration. If periodic deformations were realized in such samples, the stripes would probably be poorly visible.

References

[1] BARBERO, G., and BARBERI, R., 1983, *J. Phys. (Fr.)*, **44,** 609.
[2] LAVRENTOVICH, O. D., and PERGAMENSHCHIK, V. M., 1990, *Mol. Cryst. liq. Cryst.*, **179,** 125.
[3] LONBERG, F., and MEYER, R., 1985, *Phys. Rev. Lett.*, **55,** 718.
[4] CLADIS, P., and TORZA, S., 1975, *J. appl. Phys.*, **46,** 584.
[5] CHIGRINOV, V. G., BELYAEV, V. V., BELYAEV, S. V., and GREBENKIN, M. F., 1979, *Zh. exp. teor. Fiz.*, **77,** 2081 [1979, *Sov. Phys. JETP*., **50,** 994].
[6] LAVRENTOVICH, O. D., 1991, *Phys. Scr.*, **T39,** 394.
[7] PERGAMENSHCHIK, V. M., 1993, *Phys. Rev. E*, **47,** 1881.
[8] LAVRENTOVICH, O. D., and PERGAMENSHCHIK, V. M., 1994, *Phys. Rev. Lett.*, **73,** 979.
[9] SPARAVIGNA, A., KOMITOV, L., STEBLER, B., and STRIGAZZI, A., 1991, *Mol. Cryst. liq. Cryst.*, **207,** 265.
[10] SPARAVIGNA, A., KOMITOV, L., and STRIGAZZI, A., 1992, *Mol. Cryst. liq. Cryst.*, **212,** 289.
[11] SPARAVIGNA, A., and STRIGAZZI, A., 1992, *Mol. Cryst. liq. Cryst.*, **221,** 109.
[12] SPARAVIGNA, A., KOMITOV, L., LAVRENTOVICH, O. D., and STRIGAZZI, A., 1992, *J. Phys. II (Fr.)*, **2,** 1881.
[13] SPARAVIGNA, A., KOMITOV, L., PALFFY-MUHORAY, P., and STRIGAZZI, A., 1993, *Liq. Cryst.*, **14,** 1945.
[14] SPARAVIGNA, A., LAVRENTOVICH, O. D., and STRIGAZZI A., 1994, *Phys. Rev. E*, **49,** 1344.
[15] KRZYŻAŃSKI, D., and DERFEL, G., 2001, *Phys. Rev. E*, **63,** 021 702.
[16] KRZYŻAŃSKI, D., and DERFEL, G., 2002, *Proc. SPIE*, **4759,** 237.
[17] KRZYŻAŃSKI, D., and DERFEL, G., 2000, *Phys. Rev. E*, **61,** 6663.
[18] BRADSHAW, M. J., RAYNES, E. P., BUNNING, J. D., and FABER, T. E., 1985, *J. Phys. (Fr.)*, **46,** 1513.
[19] FAETTI, S., GATTI, G., and PALLESCHI, V., 1985, *J. Phys. Lett. (Fr.)*, **46,** L-881.
[20] ERICKSEN, J. L., 1966, *Phys. Fluids*, **9,** 1205.
[21] YOKOYAMA, H., 1997, *Phys. Rev. E*, **55,** 2938.
[22] FAETTI, S., 1987, *Phys. Rev. A*, **36,** 408.
[23] YOKOYAMA, H., 1988, *Mol. Cryst. liq. Cryst.*, **165,** 265.
[24] KRZYŻAŃSKI, D., and DERFEL, G., 2002, *Liq. Crys.*, **29,** 951.
[25] SCHILLER, P., 1990, *Phase Transitions*, **29,** 59.
[26] YARIV, A., and YEH, P., 1984, *Optical Waves in Crystals* (New York: Wiley).